Stack-Sorting Diagrams and Tableaux

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South Pasadena High School

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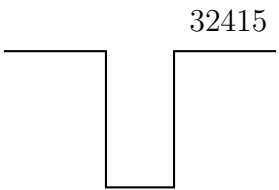

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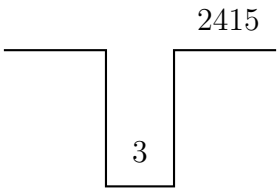

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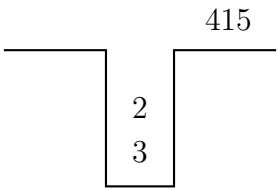

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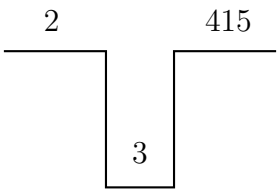

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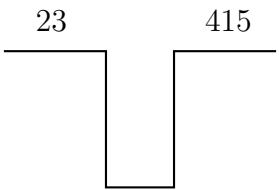

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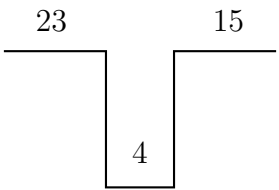

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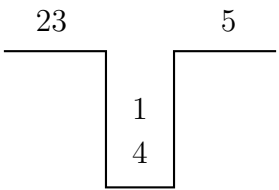

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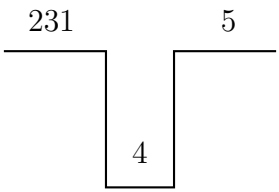

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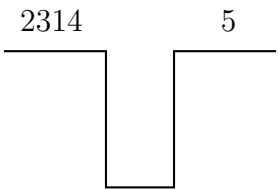

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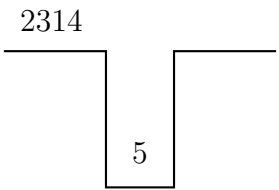

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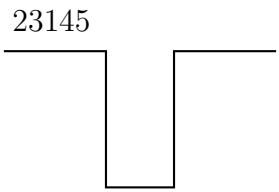

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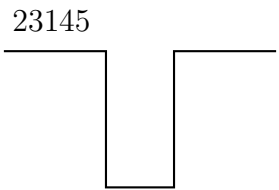

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Try it for yourself: $s(314592687) = 134526789$.

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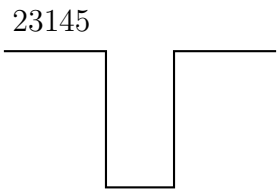

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Observation. If a permutation π is of length n, then $s^{n-1}(\pi)$ is sorted.

We denote the minimum integer k for which $s^k(\pi)$ is sorted by $\langle\pi\rangle$, and a permutation π is *t-stack-sortable* if and only if $\langle\pi\rangle \leq t$.

Some Interesting Questions

Let $\mathcal{W}_t(n)$ denote the set of all t-stack-sortable permutations in S_n. What is $|\mathcal{W}_t(n)|$?

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1	$C_n = \frac{1}{n+1}\binom{2n}{n}$	Knuth (1968)
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Theorem (West 1990)

A permutation is 1-stack-sortable if and only if it avoids the pattern 231. A permutation is 2-stack-sortable if and only if it avoids 2341 and 3241 patterns that are not part of a greater 35241 pattern.

Some Interesting Questions

For all $n \in \mathbb{N}$, let

$$\mathcal{D}_n := \frac{1}{n!} \sum_{\pi \in S_n} \langle \pi \rangle.$$

What is the asymptotic behavior of $\mathcal{D}_n / n$ as $n \to \infty$?

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Theorem (Defant 2020)

$$0.62433 \approx \lambda \leq \liminf_{n\to\infty} \frac{\mathcal{D}_n}{n} \leq \limsup_{n\to\infty} \frac{\mathcal{D}_n}{n} \leq \frac{3}{5}(7 - 8\ln 2) \approx 0.87289$$

Introducing S'_n

Let $S'_n := \{\rho 0 \mid \rho \in S_n\}$ and

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If $0 \in \pi$, then let $C_\pi = (c_{\pi,j})_{\geq 1}$ denote the sequence of right-to-left maxima of π to the left of 0. For all $\pi \in S'_n$, we write

$$\begin{aligned} s(\pi) &= s\left(b_{\pi,1} c_{\pi,1} b_{\pi,2} c_{\pi,2} \dots b_{\pi,|C_\pi|} c_{\pi,|C_\pi|} 0\right) \\ &= s(b_{\pi,1}) s(b_{\pi,2}) \dots s(b_{\pi,|C_\pi|}) 0 \underbrace{c_{\pi,|C_\pi|} \dots c_{\pi,1}}_{\text{increasing tail}}. \end{aligned}$$

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Proposition (Defant 2020)

$$\lim_{n \to \infty} \left(\frac{\mathcal{D}_n}{n} - \frac{\mathcal{D}'_n}{n} \right) = 0$$

Example: $\pi = 9\ 3\ 10\ 7\ 8\ 2\ 6\ 1\ 4\ 5\ 0$

(9 3) 10 (7) 8 (2) 6 (1 4) 5 0

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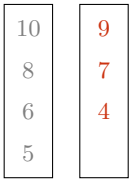
$(9\ 3)\ 10\ (7)\ 8\ (2)\ 6\ (1\ 4)\ 5\ 0$

$\downarrow s$

$(3)\ 9\ 7\ (2\ 1)\ 4\ 0\ 5\ 6\ 8\ 10$



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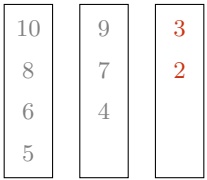
$$\downarrow s$$

$$(3)\ 9\ 7\ (2\ 1)\ 4\ 0\ 5\ 6\ 8\ 10$$

$$\downarrow s$$

$$3\ (1)\ 2\ 0\ 4\ 5\ 6\ 7\ 8\ 9\ 10$$



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(3) 9 7 (2 1) 4 0 5 6 8 10

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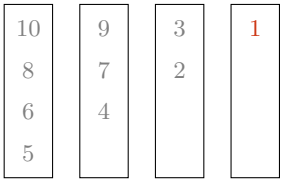
3 (1) 2 0 4 5 6 7 8 9 10

$\downarrow s$

1 0 2 3 4 5 6 7 8 9 10



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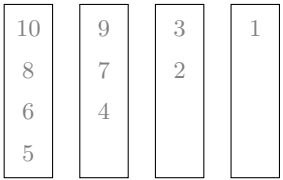

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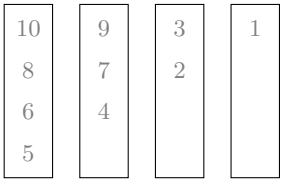

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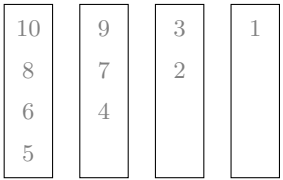

Note that there are $\langle \pi \rangle = 4$ columns.
How do we arrange the rows?

Towards the Construction of the Stack-Sorting Tableaux

Fix any permutation $\pi \in S'_n$. For all $i \in [n]$, let the *column* $\mathsf{col}_\pi(i)$ be the pass on which i is added to the tail. Furthermore, if $\mathsf{col}_\pi(i) = 1$, then let the *row* $\mathsf{row}_\pi(i)$ be the index of i in $C_{\pi,1}$, and otherwise, let it be the same as the row of the first entry to the right of i in $s^{\mathsf{col}_\pi(i)-2}(\pi)$ that is in column $\mathsf{col}_\pi(i) - 1$.

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10
8
6
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3 9 7 2 1 4 0 5 6 8 10

10	9
8	7
6	
5	4

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9 3 10 7 8 2 6 1 4 5 0

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3 9 7 2 1 4 0 5 6 8 10

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3 1 2 0 4 5 6 7 8 9 10

10	9	3
8	7	
6		
5	4	2

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$$
\begin{array}{c}
9\ 3\ 10\ 7\ 8\ 2\ 6\ 1\ 4\ 5\ 0 \\
\downarrow s \\
3\ 9\ 7\ 2\ 1\ 4\ 0\ 5\ 6\ 8\ 10 \\
\downarrow s \\
3\ 1\ 2\ 0\ 4\ 5\ 6\ 7\ 8\ 9\ 10 \\
\downarrow s \\
1\ 0\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10
\end{array}
\qquad
\begin{array}{|c|c|c|c|}
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8 & 7 & & \\ \hline
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\end{array}
$$

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For every permutation $\pi \in S'_n$, define the composition $\alpha_\pi \models n$ such that the jth part of α_π is the number of entries i satisfying $\mathsf{row}_\pi(i) = j$. In our running example, $\alpha_\pi = (3, 2, 1, 4)$.

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For every composition α, define the *composition diagram*

$$D(\alpha) := \{(i, j) \in \mathbb{N}^2 : i \leq \alpha_j\}.$$

We impose a partial order relation $\geq_D$ on $D(\alpha)$ such that $(x, y) \geq_D (z, w)$ if and only if $x \leq z$ and $y \leq w$. We refer to $D(\alpha_\pi)$ as the *stack-sorting diagram* of π.

Stack-Sorting Diagrams and Tableaux

For every permutation $\pi \in S'_n$, define the composition $\alpha_\pi \models n$ such that the jth part of α_π is the number of entries i satisfying $\mathsf{row}_\pi(i) = j$. In our running example, $\alpha_\pi = (3, 2, 1, 4)$.

For every composition α, define the *composition diagram*

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We impose a partial order relation $\geq_D$ on $D(\alpha)$ such that $(x, y) \geq_D (z, w)$ if and only if $x \leq z$ and $y \leq w$. We refer to $D(\alpha_\pi)$ as the *stack-sorting diagram* of π.

Now, define the function $T_\pi : [n] \to D(\alpha_\pi)$ such that

$$T_\pi(i) := (\mathsf{col}_\pi(i), \mathsf{row}_\pi(i))$$

for all $i \in [n]$. We refer to T_π as the *stack-sorting tableau* of π.

Stack-Sorting Diagrams and Tableaux

Theorem (Zhang 2026)

For every permutation $\pi \in S'_n$, the function T_π is a linear extension of the poset $(D(\alpha_\pi), \geq_D)$.

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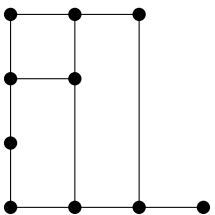

Hasse diagram of $D(3, 2, 1, 4)$

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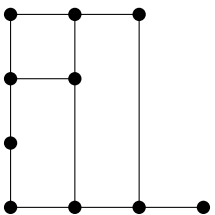
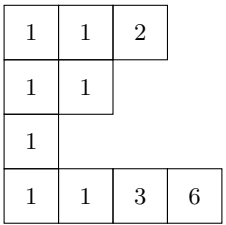


Hasse diagram of $D(3, 2, 1, 4)$

But how is this useful?

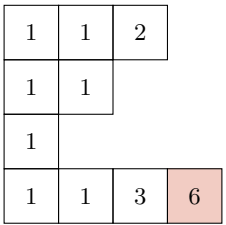
Hook-Length Formula for Stack-Sorting Tableaux

Example:



Hook-Length Formula for Stack-Sorting Tableaux

Example: $h_\alpha(4,4)$ for $\alpha = (3,2,1,4)$



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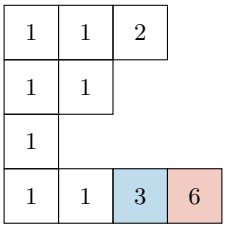

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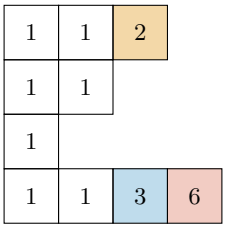

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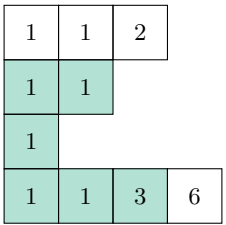

Hook-Length Formula for Stack-Sorting Tableaux

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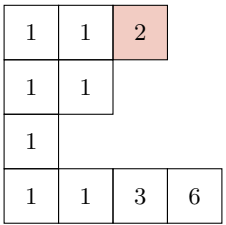

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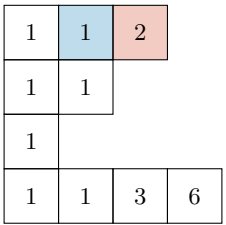

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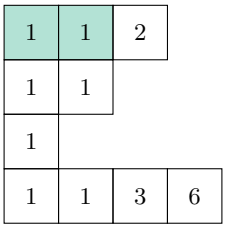

Hook-Length Formula for Stack-Sorting Tableaux

Theorem (Zhang 2026)

For every linear extension T of a composition $\alpha \models n$, the number of permutations $\pi \in S'_n$ satisfying $T_\pi = T$ is

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Notice how the above formula solely depends on α, so the number of permutations $\pi \in S'_n$ satisfying $\alpha_\pi = \alpha$ is

$$\mathcal{L}(D(\alpha)) \cdot \prod_{(i,j)\in D(\alpha)} h_\alpha(i,j)$$

where $\mathcal{L}(D(\alpha))$ denotes the number of linear extensions of $D(\alpha)$.

Two Applications

Theorem (Zhang 2026)

Fix $t \in \mathbb{N}$. For all $n \in \mathbb{N}$, the number of t-stack-sortable permutations in S'_n is computable in $O(n^{3t+1})$ time.

Two Applications

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Main idea: For every composition α, we can compute the number of permutations π such that $\alpha_\pi = \alpha$, and the stack-sorting depth of π is precisely the width of α. The rest follows from grouping compositions together based on shape statistics and doing clever dynamic programming.

Two Applications

Theorem (Zhang 2026)

The limit of $\mathcal{D}_n/n$ exists, and $(\mathcal{D}'_{n-1}/n)_{n=2}^{\infty}$ is a monotonically increasing sequence that converges to $\lim_{n\to\infty} \mathcal{D}_n/n$.

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Main idea: We can reformulate $\mathcal{D}'_n$ in the language of stack-sorting tableaux through algebraic manipulation.

$$0.73286 \approx \frac{\mathcal{D}'_{45}}{46} \leq \lim_{n\to\infty} \frac{\mathcal{D}_n}{n} \leq \frac{3}{5}(7 - 8\ln 2) \approx 0.87289.$$

Thank you!

Questions?

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