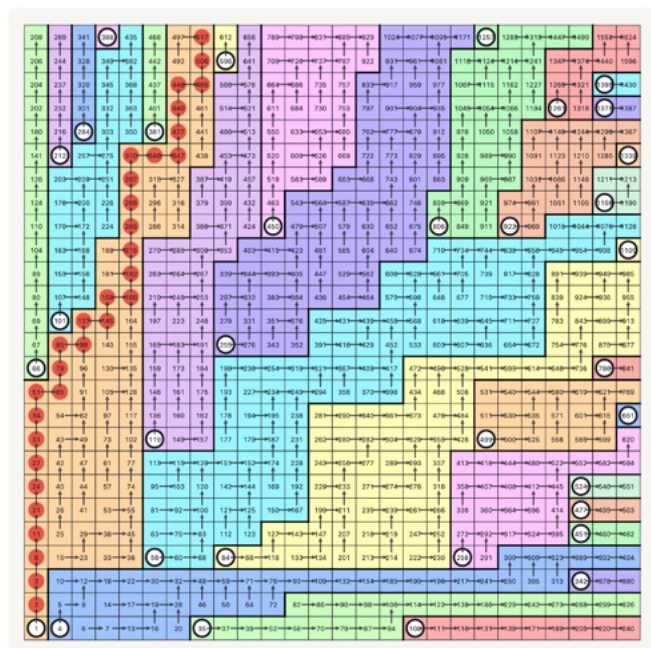
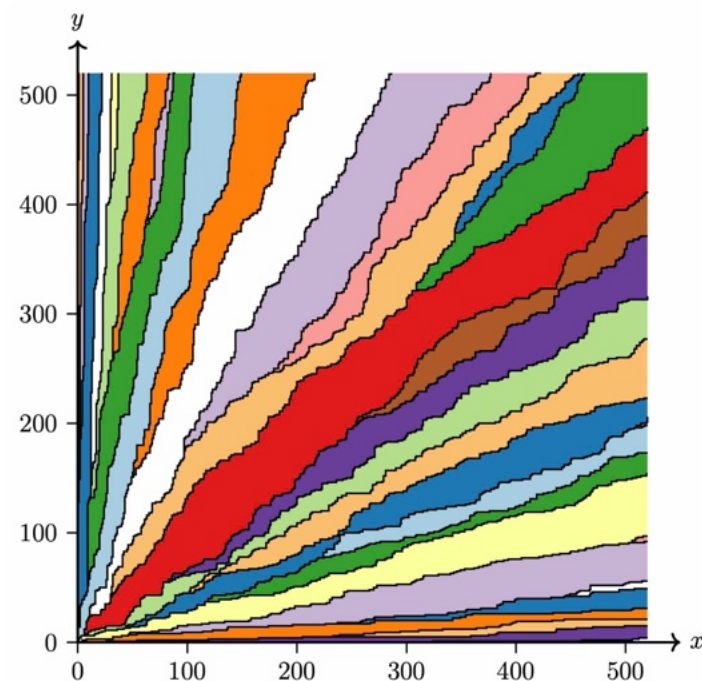


An animated introduction to Plancherel measure: from longest increasing subsequences to jeu de taquin forests on infinite Young tableaux

Dan Romik (UC Davis)

Permutation Patterns 2026

Cal State Los Angeles, July 22, 2026



Talk plan

- **Old math:** longest increasing sequences; the Ulam-Hammersley problem; Young tableaux and Schensted's theorem; the RSK correspondence; Plancherel growth process; Plancherel measure; Jeu de taquin
- **New-ish math:** Jeu de taquin dynamics (Romik-Śniady 2015)
- **New math:** Jeu de taquin forests (Romik-Śniady 2026)
- **Software demo:** understanding math through interactive visualization

Based on joint work with Piotr Śniady

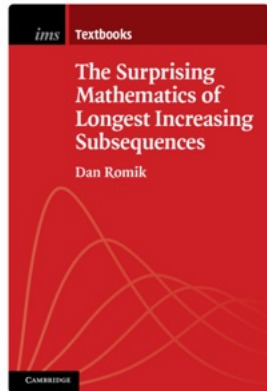
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Source material for this talk

- D. Romik. *The Surprising Mathematics of Longest Increasing Subsequences*. Cambridge University Press, 2015. (Covers the material in the first half of the talk, and much more.)



- D. Romik, P. Śniady. Jeu de taquin dynamics on infinite Young tableaux and second class particles. *Ann. Probab.* 43 (2015), 682-737.
- D. Romik, P. Śniady. Jeu de taquin forests and the inverse infinite RSK correspondence. Preprint, 2026. (Available from danromik.com/pages/papers.html.)
- RSK Explorer software. (Available from danromik.com/pages/software.html.)

Longest increasing subsequences

Let $\sigma = (\sigma(1), \dots, \sigma(N))$ be a permutation of order N .

- An **increasing subsequence** of σ is a subsequence

$$\sigma(i_1), \sigma(i_2), \dots, \sigma(i_k) \quad (i_1 < i_2 < \dots < i_k)$$

whose values are increasing: $\sigma(i_1) < \sigma(i_2) < \dots < \sigma(i_k)$.

- Define

$L(\sigma) =$ the maximal length of an increasing subsequence of σ .

Example. For $\sigma = (3, 1, 6, 7, 2, 5, 4)$ we have $L(\sigma) = 3$: for example, $(3, 6, 7)$ is a longest increasing subsequence (there are others).

The Ulam-Hammersley problem

Let σ_N denote a uniformly random permutation of order N , and denote

$$L_N = L(\sigma_N), \quad \ell_N = \mathbb{E}(L_N).$$

Question (Ulam, 1961). What is the asymptotic behavior of ℓ_N as $N \rightarrow \infty$?

- Ulam experimented with Monte Carlo simulation, which suggested that ℓ_N grows like $c\sqrt{N}$ for some constant c .
- **Theorem (Hammersley, 1972).** The limit

$$c = \lim_{N \rightarrow \infty} \frac{\ell_N}{\sqrt{N}}$$

exists (and moreover $L_N/\sqrt{N} \rightarrow c$ in probability).

(Key proof ideas: **Poissonization** and **subadditivity**.)

- The problem of determining c became known as the **Ulam-Hammersley problem**. Simulations and heuristic arguments suggested $c = 2$.

Why is $\sqrt{n}$ the correct order of magnitude?

Lemma 1. $\ell_n \geq \sqrt{n}$ for all n .

Let $D(\sigma)$ denote the length of the longest *decreasing* subsequence of σ . The lemma is a straightforward consequence of the classical Erdős-Szekeres theorem.

Theorem (Erdős-Szekeres, 1935). For every permutation σ of order n ,

$$L(\sigma) \cdot D(\sigma) \geq n.$$

Proof. For each position i , let a_i (resp. b_i) be the length of the longest increasing (resp. decreasing) subsequence *ending* at i . If $i < j$ then $\sigma(i) < \sigma(j)$ forces $a_i < a_j$, while $\sigma(i) > \sigma(j)$ forces $b_i < b_j$; either way $(a_i, b_i) \neq (a_j, b_j)$. So $i \mapsto (a_i, b_i)$ is injective into $\{1, \dots, L(\sigma)\} \times \{1, \dots, D(\sigma)\}$, giving $n \leq L(\sigma) D(\sigma)$. ■

Proof of Lemma 1. Reversing a permutation exchanges increasing and decreasing subsequences, so L_n and $D_n := D(\sigma_n)$ have the same distribution; in particular $\mathbb{E}(L_n) = \mathbb{E}(D_n)$. Using Erdős-Szekeres and AM-GM, $\frac{L_n + D_n}{2} \geq \sqrt{L_n D_n} \geq \sqrt{n}$ pointwise, hence

$$\ell_n = \mathbb{E}(L_n) = \frac{1}{2} \mathbb{E}(L_n + D_n) \geq \sqrt{n}. \quad \blacksquare$$

Why is $\sqrt{n}$ the correct order of magnitude?

Lemma 2. $\ell_n \leq (e + o(1))\sqrt{n}$ as $n \rightarrow \infty$.

Proof. The number $X_{n,k}$ of increasing subsequences of length k in σ_n has expected value

$$\mathbb{E}(X_{n,k}) = \binom{n}{k} \cdot \frac{1}{k!},$$

since there are $\binom{n}{k}$ choices of positions, and the k values in those positions have probability $1/k!$ to be in increasing order. By Markov's inequality,

$$\text{Prob}(L_n \geq k) \leq \mathbb{E}(X_{n,k}) = \binom{n}{k} \frac{1}{k!} \leq \frac{n^k}{(k!)^2}.$$

Taking $k \approx t\sqrt{n}$ and using the weak form $k! \geq (k/e)^k$ of Stirling's approximation, this gives

$$\text{Prob}(L_n \geq t\sqrt{n}) \leq \frac{n^k}{(k/e)^k} = \left(\frac{ne^2}{k^2}\right)^k \approx \left(\frac{e^2}{t^2}\right)^{t\sqrt{n}}.$$

This bound goes to 0 as $n \rightarrow \infty$ provided $t > e$. ■

Young diagrams and Young tableaux

- A **partition** of n is a sequence $\lambda = (\lambda_1, \dots, \lambda_k)$ of integers with $\lambda_1 \geq \dots \geq \lambda_k \geq 1$ and $\lambda_1 + \dots + \lambda_k = n$. We represent λ graphically as a **Young diagram**: k left-justified rows of boxes, with λ_i boxes in the i -th row. We write $|\lambda| = n$ (the **size** of λ).
- A **Young tableau** of shape λ , with entries from a set of n distinct numbers, is a filling of the boxes of λ with those numbers that is increasing along rows and columns.
- A **standard Young tableau (SYT)** is the case where the entries are $1, \dots, n$.
- The number f_λ of SYTs of shape λ is given by the **hook length formula** (Frame-Robinson-Thrall, 1954):

$$f_\lambda = \frac{n!}{\prod_{b \in \lambda} h(b)},$$

where $h(b)$, the **hook length** of the box b , is the number of boxes to the right of b in its row, plus the number of boxes below b in its column, plus 1.

Example: $\lambda = (4, 4, 2, 1)$

the Young diagram

1	3	4	7
2	5	8	10
6	9		
11			

a standard Young tableau

7	5	3	2
6	4	2	1
3	1		
1			

hook lengths

By the hook length formula,

$$f_\lambda = \frac{11!}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 3 \cdot 2 \cdot 2 \cdot 1 \cdot 1 \cdot 1} = \frac{39916800}{30240} = 1320.$$

The RSK correspondence

Given distinct numbers $z_1, \dots, z_n$, the **RSK correspondence** (aka **Robinson-Schensted-Knuth correspondence**, aka **Robinson-Schensted correspondence**) is a map

$$(z_1, \dots, z_n) \mapsto (\lambda, P, Q)$$

where

λ is a Young diagram of size n

P is a Young tableau of shape λ with entries $z_1, \dots, z_n$ (the **insertion tableau**)

Q is a Young tableau of shape λ with entries $1, \dots, n$ (the **recording tableau**)

This map is a bijection: given the triple (λ, P, Q) we can recover the sequence $z_1, \dots, z_n$ by inverting the steps of the algorithm.

Schensted's theorem

The connection between RSK and longest increasing subsequences:

Theorem (Schensted, 1961). If $(\lambda, P, Q) = \text{RSK}(z_1, \dots, z_n)$, then

$$\lambda_1 = \text{the length of the longest increasing subsequence of } (z_1, \dots, z_n).$$

- Dually, the length of the first **column** of λ is the length of the longest **decreasing** subsequence.
- **Consequence:** $L(\sigma_N)$ is equal in distribution to λ_1 , the first row length of the RSK shape λ of a uniformly random permutation of order N .
- So, to solve the Ulam-Hammersley problem we should understand the distribution of the RSK shape of a random permutation. (This seems more difficult than just understanding how the first row behaves, but somehow ends up being the correct question to ask.)

Finite Plancherel measure

When we apply the RSK correspondence to the numbers $1, \dots, n$, arranged in some order, (i.e., a permutation of order n), the output is a triple (λ, P, Q) in which λ is a Young diagram of size n , and P and Q are both **standard Young tableaux** (SYTs), that is, Young tableaux with entries $1, \dots, n$.

If the permutation is a uniformly random permutation of order n , the probability distributions of λ, P, Q are given by

$$\text{Prob}(P = T) = \frac{f_\lambda}{n!},$$

$$\text{Prob}(Q = T) = \frac{f_\lambda}{n!},$$

$$\text{Prob}(\lambda = \mu) = \frac{f_\lambda^2}{n!},$$

where, as before,

$f_\lambda =$ the number of SYTs of shape λ

This measure on random Young diagrams (or on SYTs) is called the **Plancherel measure** of order n .

The Logan-Shepp-Vershik-Kerov limit shape theorem

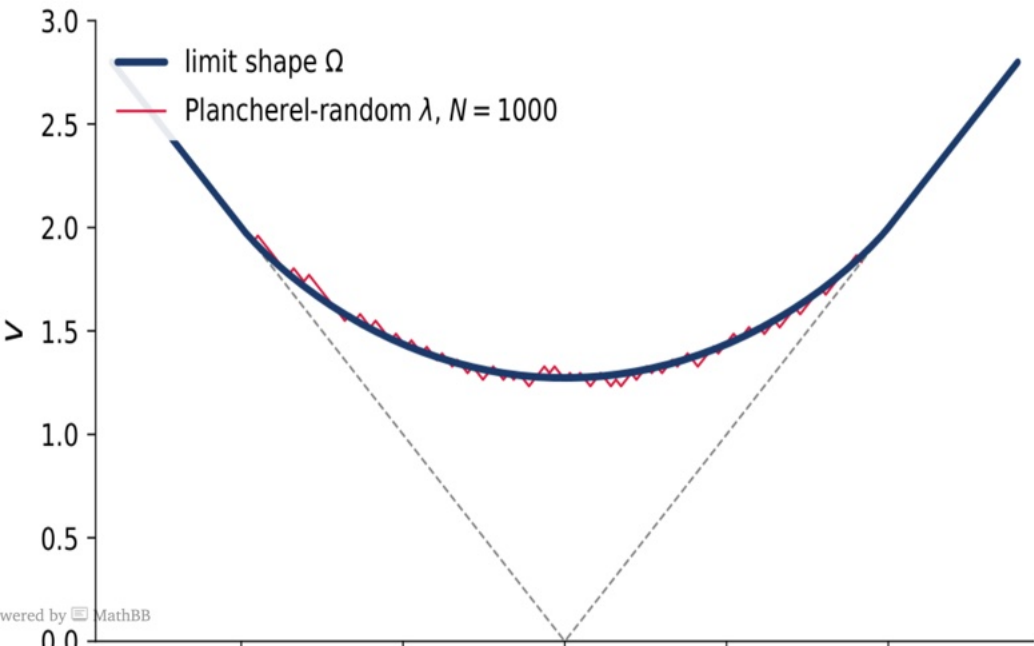
Combining Schensted's theorem with the definition of Plancherel measure:

$$L_N \stackrel{\text{Dist}}{=} \lambda_1, \quad \text{where } \lambda \text{ is a Plancherel-random Young diagram of order } N.$$

So the Ulam-Hammersley problem reduces to understanding the asymptotic behavior of Plancherel-random Young diagrams. This problem was solved in 1977 independently by Logan-Shepp and Vershik-Kerov.

Theorem (Logan-Shepp, 1977; Vershik-Kerov, 1977). Scaled down by a factor $\sqrt{N}$, Plancherel-random Young diagrams of order N converge (in probability, uniformly) to a **limit shape**. In rotated ("Russian") coordinates, the limit shape is the region bounded by the curve

$$\Omega(u) = \begin{cases} \frac{2}{\pi} \left(u \arcsin \left(\frac{u}{2} \right) + \sqrt{4 - u^2} \right) & \text{if } |u| \leq 2, \\ |u| & \text{if } |u| > 2. \end{cases}$$



The Logan-Shepp-Vershik-Kerov limit shape theorem (continued)

Corollary (solution of the Ulam-Hammersley problem). $c = 2$, that is,

$$\frac{\ell_N}{\sqrt{N}} \rightarrow 2, \quad \frac{L_N}{\sqrt{N}} \rightarrow 2 \text{ in probability.}$$

Notes

- The proofs use the **hook length formula**: maximizing f_λ becomes a variational problem for a "hook integral", whose minimizer is the curve Ω .
- The limit shape immediately gives the lower bound $c \geq 2$; the matching upper bound $c \leq 2$ required an additional argument (Vershik-Kerov).

The Baik-Deift-Johansson theorem

With the law of large numbers $L_N \approx 2\sqrt{N}$ established, the next natural question is about the **fluctuations** of L_N around $2\sqrt{N}$. This was answered 22 years later in a landmark result:

Theorem (Baik-Deift-Johansson, 1999). For all $t \in \mathbb{R}$,

$$\text{Prob} \left(\frac{L_N - 2\sqrt{N}}{N^{1/6}} \leq t \right) \xrightarrow{N \rightarrow \infty} F_2(t),$$

where $F_2(t) = \exp \left(- \int_t^\infty (x - t) q(x)^2 dx \right)$, and q is the Hastings-McLeod solution of the **Painlevé II** equation $q'' = tq + 2q^3$.

Notes

- F_2 is the **Tracy-Widom distribution** from random matrix theory: the limiting distribution of the (scaled) largest eigenvalue of a random matrix from the Gaussian Unitary Ensemble (GUE)!
- Extensions (Baik-Deift-Johansson, Okounkov, Borodin-Okounkov-Olshanski, Johansson, 1999-2000): the k -th row λ_k of a Plancherel-random diagram fluctuates like the k -th largest GUE eigenvalue.

Plancherel growth process

So far we studied random permutations of a fixed order n . The second half of the talk is about a way to couple all the finite Plancherel measures into a single infinite probabilistic object.

Let $X_1, X_2, \dots$ be an infinite sequence of i.i.d. random variables with the $U(0, 1)$ distribution. For each n , denote by $(\lambda^{(n)}, P_n, Q_n)$ the output of the RSK correspondence applied to $(X_1, \dots, X_n)$. Then $(\lambda^{(n)}, P_n, Q_n)$ is distributed according to Plancherel measure of order n .

In other words, this mapping defines a natural coupling of all the finite Plancherel measures, known as the **Plancherel growth process**.

Infinite Plancherel measure and infinite Young tableaux

The evolution of the Young diagrams $\lambda^{(n)}$ in the Plancherel growth process is tracked through the sequence of recording tableaux Q_n , which has a natural limit, a random **infinite Young tableau** $Q = Q_\infty$. The distribution of this infinite Young tableau is called (infinite) **Plancherel measure**.

We can view Q as the output of the **infinite RSK map** acting on the infinite sequence $X_1, X_2, \dots$:

$$Q = \text{RSK}(X_1, X_2, X_3, \dots)$$

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Notes

- In this limiting picture, there is no insertion tableau P anymore - the sequence of insertion tableaux P_n does not have a meaningful limit.
- A natural question: is this infinite RSK map invertible? The fact that we lose the insertion tableau suggests that the answer may be "no". Is this intuition correct?

Jeu de taquin

Jeu de taquin (JDT) is an important combinatorial operation that was introduced by M.-P. Schützenberger in 1977, originally in the context of finite Young tableaux. It has deep connections to the RSK correspondence.

In our setting of the infinite tableau Q , we can define the **jeu de taquin path** as a monotone (up-right) sequence of boxes that starts at the corner box $(1, 1)$ and successively moves one step up or to the right towards the box containing the smaller of the two entries.

The **jeu de taquin map** is a function

$$J : Q \mapsto J(Q)$$

that takes the tableau Q and performs the following operations:

- Remove the box at $(1, 1)$.
- Slide each box after $(1, 1)$ in the jeu de taquin path to the box occupied by the previous position in the path.
- Decrease all entries in the tableau by 1

Properties of the jeu de taquin map

In our 2015 paper "Jeu de taquin dynamics on infinite Young tableaux and second class particles", Piotr Śniady and I proved various results about the interaction between $\text{RSK}(\cdot)$, $J(\cdot)$, and Plancherel measure.

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1. The jeu de taquin map preserves Plancherel measure:

$$J(Q) \stackrel{\text{Dist}}{=} Q$$

In other words, if $(\Omega, \mathcal{F}, P)$ denotes the measure space of infinite Young tableaux with the Plancherel measure, then $(\Omega, \mathcal{F}, P, J)$ is a measure preserving system.

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2. The map RSK is a **factor map** (aka **homomorphism**) from the measure preserving system

$$([0, 1]^{\mathbb{N}}, \mathcal{B}, \Lambda^{\mathbb{N}}, S)$$

(where $\Lambda^{\mathbb{N}}$ is the product of Lebesgue measures on $(0, 1)$, and S is the shift map: $S(z_1, z_2, \dots) = (z_2, z_3, \dots)$) to the measure preserving system

$$(\Omega, \mathcal{F}, P, J).$$

More concretely, this means that the relation

$$J \circ \text{RSK} = \text{RSK} \circ S$$

holds almost surely with respect to the measure $\Lambda^{\mathbb{N}}$.

Properties of the jeu de taquin map (continued)

3. The map RSK is an **isomorphism** between the two measure preserving systems. That is, the inverse map RSK^{-1} exists almost surely. More explicitly, there exists a measurable function on $(\Omega, \mathcal{F}, P)$ such that if

$$Q = \text{RSK}(X_1, X_2, \dots)$$

(with $X_1, X_2, \dots$ the iid sequence of $U(0, 1)$ random variables from before) then almost surely

$$\begin{aligned} X_1 &= \varphi(Q), \\ X_2 &= \varphi(J(Q)), \\ X_3 &= \varphi(J^2(Q)), \\ &\vdots \end{aligned}$$

Properties of the jeu de taquin map (continued)

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$$\begin{aligned} X_1 &= \varphi(Q), \\ X_2 &= \varphi(J(Q)), \\ X_3 &= \varphi(J^2(Q)), \\ &\vdots \end{aligned}$$

4. Moreover, $\varphi(\cdot)$ can be described explicitly by

$$\varphi(Q) = 1 - F(\text{the asymptotic direction of the JDT path of } Q)$$

for an explicit function $F(x) = \int_{-2}^x \sqrt{4 - t^2} \, dt$. (The asymptotic direction is normalized to be in $(-2, 2)$.) In particular, the asymptotic direction exists almost surely.

Properties of the jeu de taquin map (continued)

3. The map RSK is an **isomorphism** between the two measure preserving systems. That is, the inverse map RSK^{-1} exists almost surely. More explicitly, there exists a measurable function on $(\Omega, \mathcal{F}, P)$ such that if

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Notes

- Properties 1-2 in this list are elementary. Properties 3-4 are difficult theorems; the proof used ideas from the representation theory of S_n .
- Property 3 refutes the earlier intuition that information is lost when we move to the infinite limit and the insertion tableau P disappears. In fact, the map remains invertible! (In an almost-sure, probabilistic sense at least.)

A new structure: jeu de taquin forests

- The 2015 results show that one can invert the RSK map and extract the sequence $X_1, X_2, \dots$ one coordinate at a time by looking at the successive iterations $Q, J(Q), J^2(Q), \dots$ of the JDT map. For each iteration, the asymptotic direction of the jeu de taquin path gives the next element in the input sequence.
- Intuitively, it appears necessary to iterate the JDT map. But is it really?
- In recent work (in preparation, 2026) we discovered this intuition is partially false. The original tableau Q already "shows" the numbers $X_1, X_2, \dots$ - but not their exact ordering.
- To see this, we need to introduce a new combinatorial structure encoded in the tableau Q , the **jeu de taquin forest (JDTF)**.

Properties of jeu de taquin forests

Theorem (Partial inversion of the RSK map using the jeu de taquin forest). Each tree in the JDT forest has an **asymptotic direction**, which maps (via the same function F as before) to exactly one of the elements in the sequence $X_1, X_2, \dots$

In other words, we have the identity (which holds almost surely)

$$\{X_1, X_2, \dots\} = \{1 - F(\text{asymptotic direction of } T) : T \text{ is a tree in JDTF}(Q)\}.$$

Properties of jeu de taquin forests (continued)

Other results give useful insight on how the trees in the JDT forests interact with the JDT map $J(\cdot)$:

1. **Evolution of the tree at the origin.** On each application of J , the **tree at the origin** (the unique tree containing the corner box $(1, 1)$) is consumed, being fully absorbed into the surrounding trees.
2. **Evolution of peripheral trees.** The other trees ("**peripheral trees**") can only grow when J is applied, but their internal structure remains the same.
3. **Finite growth of peripheral trees.** At each iteration of J , each peripheral tree grows by only a finite number of boxes.
4. **Finite tree lifetimes.** Each tree T has a unique integer $n(T) \geq 1$ with the property that after $n(T) - 1$ applications of the map J , T becomes the tree at the origin (and consequently is consumed on the $n(T)$ -th iteration).
5. **Bijection between trees and positive integers.** The map $T \mapsto n(T)$ is a bijection between the trees in the JDT forest and the positive integers.
6. **Tree cusps.** Each tree has a unique box that is minimal with respect to the natural partial order on $\mathbb{N}^2$.
7. **Asymptotic direction of trees.** For each tree T , the asymptotic direction of T is equal to the asymptotic direction of the JDT path in $J^{n(T)-1}(Q)$.
Equivalently,

$$\begin{aligned} X_{n(T)} &= 1 - F(\text{asymptotic direction of the JDT path in } J^{n(T)-1}(Q)) \\ &= 1 - F(\text{asymptotic direction of } T) \end{aligned}$$

8. **Jeu de taquin paths approach tree boundaries infinitely often.** The jeu de taquin path visits infinitely many boxes that lie at the boundary of the tree at the origin.

Properties of jeu de taquin forests (continued)

- The proofs make use of the 2015 results: all the probability is encapsulated in what the main 2015 result says about "Plancherel-generic" tableau.
- Beyond that, it's "[just combinatorics](#)" (fun blog by Igor Pak, look it up!)
- The most fundamental fact is the "stability of trees" lemma: peripheral trees can only grow when J is applied, and therefore have an identity that can be tracked over successive applications of the map, until they are "consumed".
- Explanation: given a directed edge (say, a horizontal edge) in the tree, " $X \rightarrow Y$ "
 - if it is touched by the jeu de taquin map then it belongs to the tree at the origin. Nothing to prove.
 - the only way for the edge to change after applying J is if the jeu de taquin sliding causes the box above to change, in other words we go from

$$\begin{array}{cc} W & Z \\ X & \rightarrow Y \end{array}$$

to

$$\begin{array}{cc} W & Z \\ & \uparrow \\ & X \quad Y \end{array}$$

But the reason the original arrow points to the right is that $Y < W$, and also $W < Z$, so $Y < Z$ and this scenario can't happen.

Open problems

The new structure of jeu de taquin forests gives rise to many interesting questions.

1. **The point process of tree cusps.** Does the point process of tree cusps have a scaling limit? What does the scaling limit look like.

Conjecture: an explicit scaling limit exists, and can be described using **RSK-polar coordinates**.

2. **A random permutation of $\mathbb{N}$.** There are two natural ways to biject the set of trees T in the JDT forest to the positive integers:

a. Lifetime map: $T \mapsto n(T)$

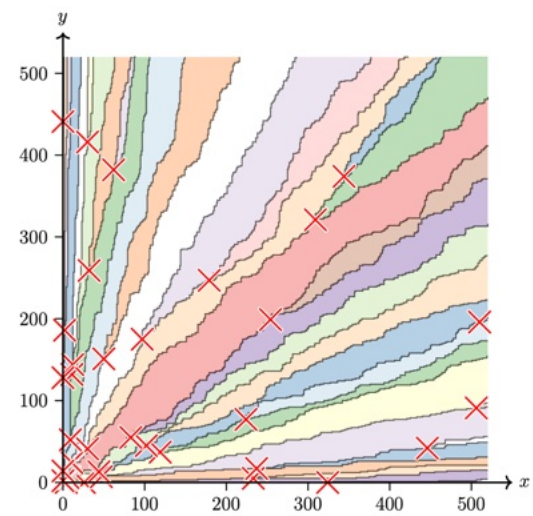
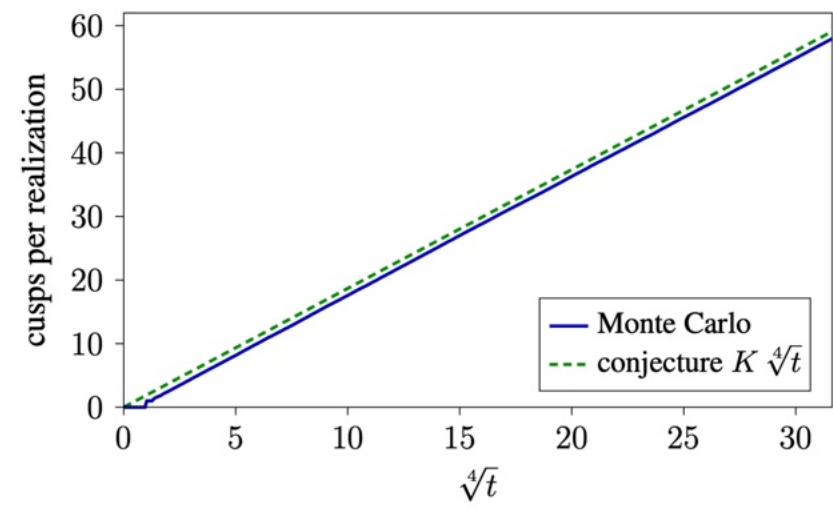
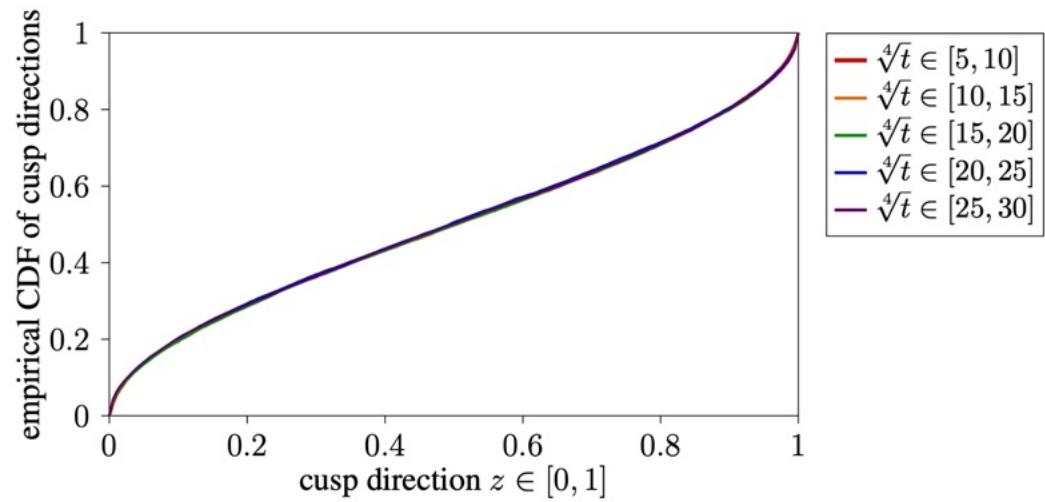
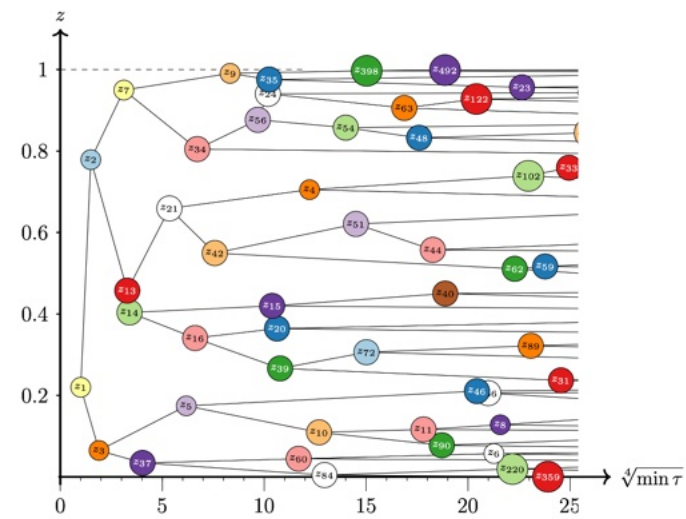
b. Entry at the tree cusp: $T \mapsto \text{ranking}(Q_{\text{cusp}(T)})$

Composing these two bijections gives a random permutation of $\mathbb{N}$ - a natural probabilistic object associated with the input sequence $X_1, X_2, \dots$

Describe its distribution, partially or fully.

3. **The tree of trees.** The trees can be arranged in the structure of a (random) complete binary tree. Describe its distribution, partially or fully.

Open problems (continued)



End notes

- ♦ Check out Piotr's postdoc ad at <https://psniady.impan.pl/index.html>

End notes

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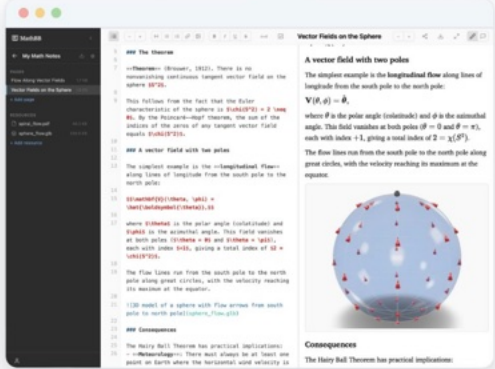
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- ✓ **Relay Chat for long-running autonomous AI research**: a customizable multi-agent chat that can work for hours on tasks you define, with configurable agents powered by Anthropic and/or OpenAI models playing assigned roles (e.g., Explorer, Prover, Skeptic).
- ✓ **GitHub integration** for backup and version history.
- ✓ **Telegram integration**: chat with Dora over Telegram to get mathematics done while on the go.



The screenshot shows a MathBB notebook workspace. On the left is a sidebar with a file explorer and a list of notebooks. The main area displays a document titled "Vector Fields on the Sphere". The document contains mathematical text and code. The text discusses the hairy ball theorem and vector fields on a sphere. A 3D visualization of a sphere with red arrows representing a vector field is shown on the right. The arrows point from the south pole towards the north pole, illustrating a longitudinal flow.

End notes

- ♦ Check out Piotr's postdoc ad at <https://psniady.impan.pl/index.html>
- ♦ This presentation was created with my AI-powered writing app MathBB. Try it out at <https://mathbb.app>. (Email me for free AI tokens.)

 **MathBB** by Rhombic Research

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Mathematics + AI = *happiness*

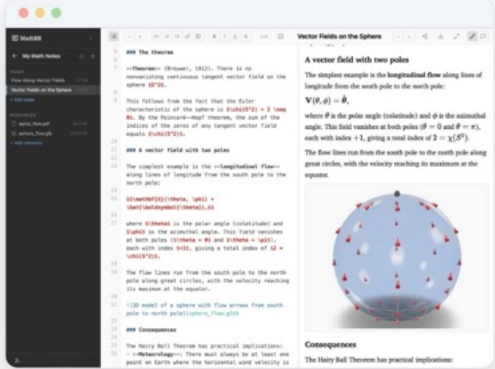
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MathBB is a notebook workspace for writing and doing mathematics: an "IDE for math".

MAIN FEATURES

- ✓ **Write in Markdown + LaTeX** with instant rendering — no compilation.
- ✓ **Draw diagrams** directly in the notebook with built-in drawing tools.
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Thank You!